Implied Probability of Default from Futures Prices Spread in Different Exchanges

Matteo Bottacini – matteo.bottacini@cryptofinance.ch

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Let's adjust the price of a future by adding a new variable: **α (alpha).**

Alpha represents the probability of Default of an Exchange. So that a higher Probability of Default will result in a higher price.

$$F_t = S_t e^{(rf_t + u - \gamma_t + \boldsymbol{\alpha_t}) * \tau}$$

Where:

$$F: Future\ price$$

$$S: Underlying\ price$$

$$rf: risk\ free\ rate$$

$$u: storage\ cost$$

$$\gamma: convenience\ yield$$

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$$\alpha: probability\ of\ default$$

$$\tau: Time\ to\ Maturity\ (T - t)$$

Then, the Spread between futures with same underlying and same time to maturity can be define as the following:

$$\frac{F_{t,1}}{F_{t,2}} = 1 + \Delta$$

Where the *1* and *2* indicates different venues.

Solving the above equation, we can find that the differential implied probability of default across two exchanges is given by:

$$\delta\alpha = \frac{\ln{(1 + \Delta)}}{\tau} - \delta u + \delta\gamma$$

In no-arbitrage conditions and under the assumptions that there are no frictions in the markets, meaning that Storage Costs and Convenience Yields among market participants in different venues are the same. Then, if the Probability of Default of an exchange is 0, the Implied Probability of Default of an Exchange given that another exchange has no probability of default is given by:

$$\alpha = \frac{\ln(1 + \Delta)}{\tau}$$